Mode mismatch calculation between our mode cleaner and OPO

By comparing the area of circle and ellipse, I did the following calculation

Area enclosed by an ellipse

The standard form: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ By putting $x = r*cos\theta$, $y = r*sin\theta$, we have $r^2 = \frac{a^2 b^2}{a^2 sin^2\theta + b^2 cos^2\theta}$

so area $= 4 \times \frac{1}{2} \int_0^{\pi/2} r^2 d\theta = 2 \int_0^{\pi/2} \frac{a^2 b^2}{a^2 sin^2\theta + b^2 cos^2\theta} d\theta = 2a^2 b^2 \int_{\theta=0}^{\theta=\pi/2} \frac{d(tan\theta)}{a^2 tan^2\theta + b^2} = 2a^2 b^2 \int_0^{\infty} \frac{du}{a^2 u^2 + b^2}$

> **The result of following calculation is we have 0.13566% loss from this shape mismatch**

In [24]:
```python
import numpy as np
import matplotlib.pyplot as plt
%matplotlib
```

```
Using matplotlib backend: Qt5Agg
```

In [8]:
```python
wh = np.sqrt(1064e-9*np.sqrt(np.cos(np.arcsin(11/281.22))*0.281435-
0.281435**2)/np.pi)
wv = np.sqrt(1064e-9*np.sqrt(0.281435/np.cos(np.arcsin(11/281.22))-
0.281435**2)/np.pi)
w = np.sqrt(1064e-9*np.sqrt(0.281435-0.281435**2)/np.pi)
print((wh-w)*1e6)
```

```
-0.1039521733245028
```

In [10]:
```python
from sympy import *
x, y = symbols('x y')
sol = solve((x**2 + y**2 - 1, x**2/(wh/w) + y**2/(wv/w) - 1), x, y)
```

In [33]:
```python
theta = sol[0][1]/sol[0][0]
```

In [36]:
```python
print(theta)
```

```
1.00028314639613
```

In [17]:
```python
from scipy import integrate
```

In [66]:
```python
s_c = np.pi/4*(np.arctan(1.00028314639613)/(np.pi/2))
eli = lambda u: 0.5*(wh/w)**2*(wv/w)**2/((wh/w)**2*u**2+(wv/w)**2)
s_e = integrate.quad(eli, 0, 1.00028314639613)
```

```
In [71]: sd = s_c-s_e[0] #part of area difference
         mm = 4*sd/s_c
```

```
In [72]: print(mm*100)
```

0.1356636451490692