```
(*define the matrixes ABCD from probe waist to PD*)
abcd1 = {{1, d1}, {0, 1}};
abcd2 = {{1, 0}, {-1 / f1, 1}};
abcd3 = {{1, d2}, {0, 1}};
abcd4 = {{1, 0}, {-1 / f2, 1}};
abcd5 = {{1, d3}, {0, 1}};
ABCD = abcd5.abcd4.abcd3.abcd2.abcd1;
ABCD // MatrixForm
A1 = ABCD[[1, 1]];
B1 = ABCD[[1, 2]];
C1 = ABCD[[2, 1]];
D1 = ABCD[[2, 2]];
(* define the matrixes ABCD from probe waist to PD through the tama-sample*)
abcd00tama = {{1, 0}, {0, 1 / n2}};
abcd01tama = {{1, thickn}, {0, 1}};
abcd02tama = {{1, 0}, {0, n2}};
abcd1tama = {{1, d1tama - thickn}, {0, 1}};
abcd2tama = {{1, 0}, {-1 / f1, 1}};
abcd3tama = {{1, d2}, {0, 1}};
abcd4tama = {{1, 0}, {-1 / f2, 1}};
abcd5tama = {{1, d3}, {0, 1}};
ABCDtama =
  abcd5tama.abcd4tama.abcd3tama.abcd2tama.abcd1tama.abcd02tama.abcd01tama.abcd00tama;
ABCDtama // MatrixForm
A1tama = ABCDtama[[1, 1]];
B1tama = ABCDtama[[1, 2]];
C1tama = ABCDtama[[2, 1]];
D1tama = ABCDtama[[2, 2]];

(*define the initial q parameter, at waist*)
q0 = I * Pi * w0 ^ 2 / lambda0;
(*calculate the final q parameter, at PD*)
qPD = (A1 * q0 + B1) / (C1 * q0 + D1);
(* calculate the final q parameter, at PD, in the case of tama-sapphire*)
qPDtama = (A1tama * q0 + B1tama) / (C1tama * q0 + D1tama);
```

```
(*extract waist and RoC from qPD parameter*)
wPD = FullSimplify[1 / Sqrt[-ComplexExpand[Im[1 / qPD]] / lambda0 * Pi ]];
RPD = FullSimplify[1 / ComplexExpand[Re[1 / qPD]]];
(*extract waist and RoC from qPD parameter, in the case of tama-sapphire*)
wPDtama = FullSimplify[1 / Sqrt[-ComplexExpand[Im[1 / qPDtama]] / lambda0 * Pi ]];
RPDtama = FullSimplify[1 / ComplexExpand[Re[1 / qPDtama]]];
```

Out[7]//MatrixForm=

$$\begin{pmatrix} 1-\frac{d3+d2\left(1-\frac{d3}{f2}\right)}{f1}-\frac{d3}{f2} & d3+d2\left(1-\frac{d3}{f2}\right)+d1\left(1-\frac{d3+d2\left(1-\frac{d3}{f2}\right)}{f1}-\frac{d3}{f2}\right) \\ -\frac{1-\frac{d2}{f2}}{f1}-\frac{1}{f2} & 1+d1\left(-\frac{1-\frac{d2}{f2}}{f1}-\frac{1}{f2}\right)-\frac{d2}{f2} \end{pmatrix}$$

Out[21]//MatrixForm=

$$\begin{pmatrix} 1-\frac{d3+d2\left(1-\frac{d3}{f2}\right)}{f1}-\frac{d3}{f2} & \frac{n2\left(d3+d2\left(1-\frac{d3}{f2}\right)+\left(1-\frac{d3+d2\left(1-\frac{d3}{f2}\right)}{f1}-\frac{d3}{f2}\right)(d1tama-thickn)\right)+\left(1-\frac{d3+d2\left(1-\frac{d3}{f2}\right)}{f1}-\frac{d3}{f2}\right)thickn}{n2} \\ -\frac{1-\frac{d2}{f2}}{f1}-\frac{1}{f2} & \frac{n2\left(1-\frac{d2}{f2}+\left(-\frac{1-\frac{d2}{f2}}{f1}-\frac{1}{f2}\right)(d1tama-thickn)\right)+\left(-\frac{1-\frac{d2}{f2}}{f1}-\frac{1}{f2}\right)thickn}{n2} \end{pmatrix}$$

```
(*solve the equations system to find the Rayleigh distance,
new waist and new waist poistion, given the radius and RoC at the PD *)
risultato = FullSimplify[Solve[RPD == Z + ZR ^ 2 / Z && ZR == Pi * neww0 ^ 2 / lambda0 &&
      wPD ^ 2 == neww0 ^ 2 * (1 + (Z / ZR) ^ 2), {ZR, neww0, Z}]];
risultatotama = FullSimplify[Solve[RPDtama == Ztama + ZRtama ^ 2 / Ztama &&
      ZRtama == Pi * neww0tama ^ 2 / lambda0 &&
      wPDtama ^ 2 == neww0tama ^ 2 * (1 + (Ztama / ZRtama) ^ 2), {ZRtama, neww0tama, Ztama}]];
{ZR, neww0, Z} = {ZR, neww0, Z} /. risultato[[1]]
{ZRtama, neww0tama, Ztama} = {ZRtama, neww0tama, Ztama} /. risultatotama[[1]]
```

Out[35]=

$$\left\{\left(f1^2\,f2^2\,\text{lambda0}\,\pi\,w0^2\right)\Big/\left((f1\,(d2-f2)+d1\,(-d2+f1+f2))^2\,\text{lambda0}^2+(-d2+f1+f2)^2\,\pi^2\,w0^4\right),\right.$$
$$-\left(f1\,f2\,\text{lambda0}\,w0\right)\Big/\left(\sqrt{\left((f1\,(d2-f2)+d1\,(-d2+f1+f2))^2\,\text{lambda0}^2+(-d2+f1+f2)^2\,\pi^2\,w0^4\right)}\right),$$
$$-\left((d1\,d3\,(d2-f1)-d2\,d3\,f1-d1\,(d2+d3-f1)\,f2+(d2+d3)\,f1\,f2)\right.$$
$$(f1\,(d2-f2)+d1\,(-d2+f1+f2))\,\text{lambda0}^2+$$
$$\left.(-d2+f1+f2)\,(d2\,(d3-f2)+f1\,f2-d3\,(f1+f2))\,\pi^2\,w0^4\right)\Big/$$
$$\left.\left((f1\,(d2-f2)+d1\,(-d2+f1+f2))^2\,\text{lambda0}^2+(-d2+f1+f2)^2\,\pi^2\,w0^4\right)\right\}$$

Out[36]=
$$\left\{\left(f1^2\ f2^2\ \text{lambda0}\ n2^2\ \pi\ w0^2\right)\Big/\left(\text{lambda0}^2\ (\text{d1tama}\ (d2-f1-f2)\ n2+f1\ (-d2+f2)\ n2+(-d2+f1+f2)\ (-1+n2)\ \text{thickn})^2+(-d2+f1+f2)^2\ n2^2\ \pi^2\ w0^4\right),\ -(f1\ f2\ \text{lambda0}\ n2\ w0)\Big/\left(\sqrt{\left(\text{lambda0}^2\ (\text{d1tama}\ (d2-f1-f2)\ n2+f1\ (-d2+f2)\ n2+(-d2+f1+f2)\ (-1+n2)\ \text{thickn})^2+(-d2+f1+f2)^2\ n2^2\ \pi^2\ w0^4\right)}\right),\right.$$
$$\left(-2\ d2\ d3\ f1^2\ f2\ \text{lambda0}^2\ n2^2+d2\ f1^2\ f2^2\ \text{lambda0}^2\ n2^2+d3\ f1^2\ f2^2\ \text{lambda0}^2\ n2^2-\right.$$
$$\text{d1tama}^2\ (d2-f1-f2)\ (d3\ (-d2+f1)+(d2+d3-f1)\ f2)\ \text{lambda0}^2\ n2^2+$$
$$2\ d2\ d3\ f1^2\ \text{lambda0}^2\ n2\ \text{thickn}+4\ d2\ d3\ f1\ f2\ \text{lambda0}^2\ n2\ \text{thickn}-$$
$$2\ d2\ f1^2\ f2\ \text{lambda0}^2\ n2\ \text{thickn}-2\ d3\ f1^2\ f2\ \text{lambda0}^2\ n2\ \text{thickn}-$$
$$2\ d2\ f1\ f2^2\ \text{lambda0}^2\ n2\ \text{thickn}-2\ d3\ f1\ f2^2\ \text{lambda0}^2\ n2\ \text{thickn}+$$
$$f1^2\ f2^2\ \text{lambda0}^2\ n2\ \text{thickn}-2\ d2\ d3\ f1^2\ \text{lambda0}^2\ n2^2\ \text{thickn}-$$
$$4\ d2\ d3\ f1\ f2\ \text{lambda0}^2\ n2^2\ \text{thickn}+2\ d2\ f1^2\ f2\ \text{lambda0}^2\ n2^2\ \text{thickn}+$$
$$2\ d3\ f1^2\ f2\ \text{lambda0}^2\ n2^2\ \text{thickn}+2\ d2\ f1\ f2^2\ \text{lambda0}^2\ n2^2\ \text{thickn}+$$
$$2\ d3\ f1\ f2^2\ \text{lambda0}^2\ n2^2\ \text{thickn}-f1^2\ f2^2\ \text{lambda0}^2\ n2^2\ \text{thickn}-$$
$$2\ d2\ d3\ f1\ \text{lambda0}^2\ \text{thickn}^2+d3\ f1^2\ \text{lambda0}^2\ \text{thickn}^2-2\ d2\ d3\ f2\ \text{lambda0}^2\ \text{thickn}^2+$$
$$2\ d2\ f1\ f2\ \text{lambda0}^2\ \text{thickn}^2+2\ d3\ f1\ f2\ \text{lambda0}^2\ \text{thickn}^2-f1^2\ f2\ \text{lambda0}^2\ \text{thickn}^2+$$
$$d2\ f2^2\ \text{lambda0}^2\ \text{thickn}^2+d3\ f2^2\ \text{lambda0}^2\ \text{thickn}^2-f1\ f2^2\ \text{lambda0}^2\ \text{thickn}^2+$$
$$4\ d2\ d3\ f1\ \text{lambda0}^2\ n2\ \text{thickn}^2-2\ d3\ f1^2\ \text{lambda0}^2\ n2\ \text{thickn}^2+$$
$$4\ d2\ d3\ f2\ \text{lambda0}^2\ n2\ \text{thickn}^2-4\ d2\ f1\ f2\ \text{lambda0}^2\ n2\ \text{thickn}^2-$$
$$4\ d3\ f1\ f2\ \text{lambda0}^2\ n2\ \text{thickn}^2+2\ f1^2\ f2\ \text{lambda0}^2\ n2\ \text{thickn}^2-$$
$$2\ d2\ f2^2\ \text{lambda0}^2\ n2\ \text{thickn}^2-2\ d3\ f2^2\ \text{lambda0}^2\ n2\ \text{thickn}^2+2\ f1\ f2^2\ \text{lambda0}^2\ n2\ \text{thickn}^2-$$
$$2\ d2\ d3\ f1\ \text{lambda0}^2\ n2^2\ \text{thickn}^2+d3\ f1^2\ \text{lambda0}^2\ n2^2\ \text{thickn}^2-$$
$$2\ d2\ d3\ f2\ \text{lambda0}^2\ n2^2\ \text{thickn}^2+2\ d2\ f1\ f2\ \text{lambda0}^2\ n2^2\ \text{thickn}^2+$$
$$2\ d3\ f1\ f2\ \text{lambda0}^2\ n2^2\ \text{thickn}^2-f1^2\ f2\ \text{lambda0}^2\ n2^2\ \text{thickn}^2+$$
$$d2\ f2^2\ \text{lambda0}^2\ n2^2\ \text{thickn}^2+d3\ f2^2\ \text{lambda0}^2\ n2^2\ \text{thickn}^2-f1\ f2^2\ \text{lambda0}^2\ n2^2\ \text{thickn}^2+$$
$$d2^2\ (d3-f2)\ \text{lambda0}^2\ (f1\ n2+(-1+n2)\ \text{thickn})^2+\text{d1tama}\ \text{lambda0}^2\ n2$$
$$\left(f1\ \left(2\ d2\ d3\ (-d2+f1)+2\ (d2\ (d2+2\ d3)-(d2+d3)\ f1)\ f2+(-2\ (d2+d3)+f1)\ f2^2\right)\ n2+\right.$$
$$\left.2\ (-d2+f1+f2)\ (d2\ (d3-f2)+f1\ f2-d3\ (f1+f2))\ (-1+n2)\ \text{thickn}\right)+$$
$$\left.(-d2+f1+f2)\ (d3\ (-d2+f1)+(d2+d3-f1)\ f2)\ n2^2\ \pi^2\ w0^4\right)\Big/$$
$$\left.\left(\text{lambda0}^2\ (\text{d1tama}\ (d2-f1-f2)\ n2+f1\ (-d2+f2)\ n2+(-d2+f1+f2)\ (-1+n2)\ \text{thickn})^2+(-d2+f1+f2)^2\ n2^2\ \pi^2\ w0^4\right)\right\}$$

```
(*equals the radius at PD to find the Image
  Unit distance required for the case of tama-sample*)
distanza = Solve[wPDtama == wPD, d1tama]
```

Out[37]=
$$\left\{\left\{\text{d1tama}\to\frac{\text{d1}\ n2-\text{thickn}+n2\ \text{thickn}}{n2}\right\},\right.$$
$$\left\{\text{d1tama}\to\frac{1}{(d2\ d3-d3\ f1-d2\ f2-d3\ f2+f1\ f2)\ n2}(-\text{d1}\ d2\ d3\ n2+\text{d1}\ d3\ f1\ n2+2\ d2\ d3\ f1\ n2+\right.$$
$$\text{d1}\ d2\ f2\ n2+\text{d1}\ d3\ f2\ n2-\text{d1}\ f1\ f2\ n2-2\ d2\ f1\ f2\ n2-2\ d3\ f1\ f2\ n2-d2\ d3\ \text{thickn}+$$
$$d3\ f1\ \text{thickn}+d2\ f2\ \text{thickn}+d3\ f2\ \text{thickn}-f1\ f2\ \text{thickn}+d2\ d3\ n2\ \text{thickn}-$$
$$\left.\left.d3\ f1\ n2\ \text{thickn}-d2\ f2\ n2\ \text{thickn}-d3\ f2\ n2\ \text{thickn}+f1\ f2\ n2\ \text{thickn})\right\}\right\}$$

In[38]:=
```
vals = {f1 → 0.05, f2 → -0.00125, d1 → 0.277, d2 → 0.064, d3 → 0.075,
   lambda0 → 633 * 10^-9, w0 → 87 * 10^-6, thickn → 0.06, n2 → 1.76};
valstama = {f1 → 0.05, f2 → -0.00125, d1tama → (0.277 + 0.0259), d2 → 0.064,
   d3 → 0.075, lambda0 → 633 * 10^-9, w0 → 87 * 10^-6, thickn → 0.06, n2 → 1.76};
{ZRn, neww0n, Zn, wPD} = N[{ZR, neww0, Z, wPD} /. vals]
{ZRtaman, neww0taman, Ztaman, wPDtama} =
 Simplify[N[{ZRtama, neww0tama, Ztama, wPDtama} /. valstama]]
```

Out[40]= $\{0.000117097,\ 4.85735 \times 10^{-6},\ 0.075951,\ 0.00315056\}$

Out[41]= $\{0.000117122,\ 4.85787 \times 10^{-6},\ 0.0759509,\ 0.00315023\}$

In[48]:=
```
Simplify[-d1 + d1tama /. distanza[[1]]]
N[Simplify[-d1 + d1tama /. distanza[[1]]] /. vals]
```

Out[48]= $\dfrac{(-1 + n2)\ thickn}{n2}$

Out[49]= 0.0259091