LOSSES FROM THE CAVITY REFLECTVITY

Here we present the derivation of the formula used for computing round trip losses from the cavity reflectivity.
The normalized power (assuming $P_{\rm in} = 1$) reflected by a cavity at the resonance reads:

$$P_{\rm res} = \left[\frac{r_1 - r_2}{1 - r_1 r_2}\right]^2 \tag{1}$$

In our case the end mirror is almost completely reflecting. We can consider the losses as an increased end mirror transmissivity and we have

$$r_2 = \sqrt{1 - T_2 - L} \simeq \sqrt{1 - L} \tag{2}$$

By using Eq. 2 and some approximations we can invert Eq.1 and find

$$\boxed{L \simeq \frac{T_1}{2}\frac{1 - P_{\rm res}}{1 + P_{\rm res}}}$$

The relation between losses and change in the reflectivity for the filter cavity ($T_1 = 0.14\%$) is shown in Fig. 1

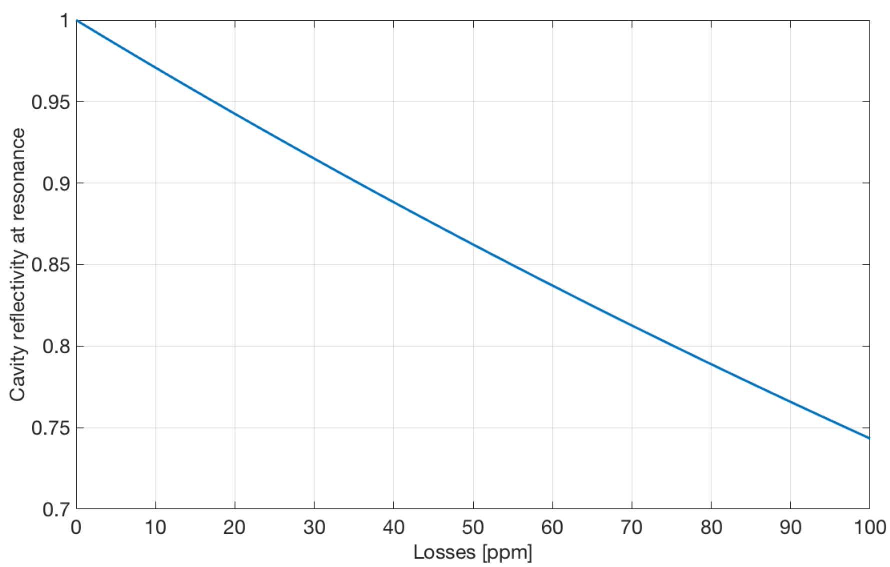

FIGURE 1. relation between losses and change in the reflectivity for the filter cavity ($T_1 = 0.14\%$)

In the following the derivation of the formula above is reported:

$$\begin{aligned}
P_{\mathrm{res}} &= \left[\frac{r_1 - \sqrt{1-L}}{1 - r_1\sqrt{1-L}}\right]^2 \\
&= \frac{r_1^2 - 2r_1\sqrt{1-L} + 1 - L}{1 - 2r_1\sqrt{1-L} + r_1^2(1-L)} \\
&\sim \frac{r_1^2 - r_1(2-L) + 1 - L}{1 - r_1(2-L) + r_1^2(1-L)} \\
&= \frac{(1-r_1)^2 - (1-r_1)L}{(1-r_1)^2 + (1-r_1)r_1 L} \\
&= \frac{(1-r_1) - L}{(1-r_1) + r_1 L}
\end{aligned}$$

where from second to third line we used $\sqrt{1-L} = 1 - L/2$.

Therefore

$$P_{\mathrm{res}}(1 - r_1 + r_1 L) = 1 - r_1 - L,$$

which means

$$L = (1 - r_1)\frac{1 - P_{\mathrm{res}}}{1 + P_{\mathrm{res}} r_1}.$$

With the approximation

$$r_1 = \sqrt{1 - T_1} \sim 1 - \frac{T_1}{2}$$

we get

$$\begin{aligned}
L &= \frac{T_1}{2}\frac{1 - P_{\mathrm{res}}}{1 + P_{\mathrm{res}} - \frac{T_1}{2}P_{\mathrm{res}}} \\
&\simeq \frac{T_1}{2}\frac{1 - P_{\mathrm{res}}}{1 + P_{\mathrm{res}}}
\end{aligned}$$